Are You Ready Chapter 2 Pretest & skills.

Attendance Problems. **Complete each sentence.**

1. __?__ points are points that lie on the same line.

2. . __?__ points are points that lie in the same plane.

3. The sum of the measures of two __?__ angles is 90°.

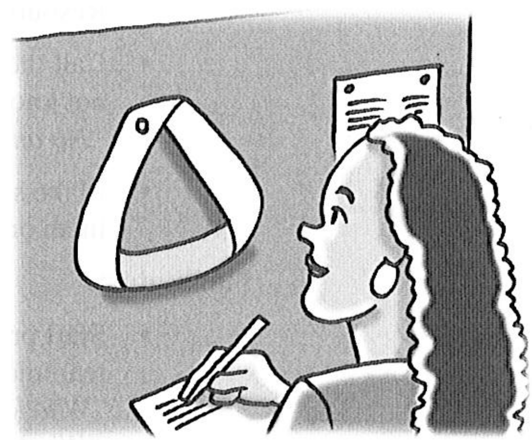
• I can use inductive reasoning to identify patterns and make conjectures.
• I can find counterexamples to disprove conjectures.

Common Core

CC.9-12.G.CO.9 Prove theorems about lines and angles.
CC.9-12.G.CO.10 Prove theorems about triangles.
CC.9-12.G.CO.11 Prove theorems about parallelograms.
CC.9-12.G.SRT.4 Prove theorems about triangles.

Vocabulary		
inductive reasoning	conjecture	counterexample

Video: Can you solve this?

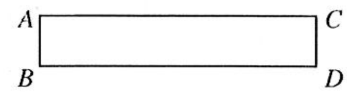


Today you will investigate what happens when you change the attributes of a Möbius strip. As you investigate, you will record data in a table. You will then analyze this data and use your results to brainstorm further experiments. As you look back at your data, you may start to consider other related questions that can help you understand a pattern and learn more about what is happening. This way of thinking, called inductive reasoning, includes not only generating new questions, but also rethinking when the results are not what you expected.

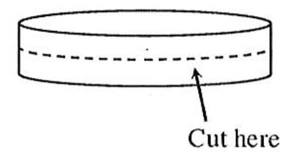
4. Working effectively with your table partners will be an important part of the learning process throughout this course. The oldest person at your table should read aloud the Table Team Expectations.

Table Team Expectations
Throughout this course, you will regularly work with your table. This collaboration will allow you to develop new ways of thinking about mathematics, increase your ability to communicate with others about math, and help you strengthen your understanding by having you explain you thinking to someone else. As you work together,
You are expected to share your ideas and contribute to the table's work.
You are expected to ask your table partners questions and to offer help to your table partners. Questions can move your table's thinking forward and help others understand ideas more clearly.
Remember that a table functions well works on the same problem together (at the same time) and discusses the problem while it works.
Remember that one student on the table should not dominate the discussion and thinking process.
Your table should regularly stop and verify that everyone on the team agrees with a suggestion or a solution.
Everyone on your table should be consulted before asking your teacher for help. Everyone should have understand the difficulty.

On the piece of newspaper provided, make a "bracelet" by taping together two ends securely together. Putting tape on both sides of the bracelet will help to make sure the bracelet is secure. In the diagram of the rectangular strip, you would tape the ends together so the point A would attach to point C, and point B would attach to point D.



5. Now predict what you think would happen if you were to cut the bracelet down the middle, as shown in the diagram. Record your prediction in the table, then actually cut the the bracelet down the middle and record the results in the last column.

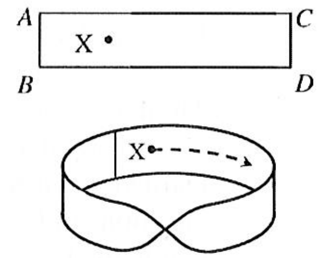


Question	Experiment	Prediction	Result
5	Cut the bracelet in half.		
6	Draw a line down the center of the Möbius strip.		
7	The Möbius strip is cut down the middle.		
8	The result from problem 7 is cut down the middle again.		
9	The Möbius strip is cut one-third the distance from the side.		

Question	Experiment	Prediction	Result
10	A strip with two twists is cut down the middle.		
11			

On a second strip of paper, label a point X in the center of the strip at least one inch away from one end.

Now turn this strip into a *Möbius strip* by attaching the ends securely together after making a twist. For the strip shown in the diagram, the paper would be twisted once so that point A would attach to point D.



A Möbius Strip

6. Predict what would happen if you were to draw a line down the center of the strip from point X until you ran out of paper.

7. What do you think would happen if you were to cut your Möbius strip along the central line you drew in problem 6? Record you predictions in your table.

Cut just one of your teams' Möbius strips. Record the results in the table. Consider the original of strip of paper from problem 5 to help you explain why cutting the Möbius strip had this result.

What else can you learn about Möbius strips? For each experiment below, first record your prediction. Then record your results in your table after conducting the experiment. Use a new Möbius strip for each experiment.

8. What if the result from problem 7 is cut in half down the middle again?

9. What would happen if the Möbius strip is cut one-third of the way from one of the sides of the strip? Be sure to cut a constant distance from the side of the strip.

10. What if a strip is formed with two twists (one full turn) instead of one? What would happen if the strip was cut down the middle?

11. Make up your own experiment. You might change how many twists you make, where to make the cuts, etc. Try to generalize your findings as you conduct your experiment. Be prepared to share your results with the class.

12. Reflection. What questions did your table partners ask that helped the team move forward? What questions do you still have about Möbius strips? What would you like to know more about?

METHODS AND MEANINGS

MATH NOTES

The Investigative Process

The **investigative process** is a way to study and learn new mathematical ideas. Mathematicians have used this process for many years to make sense of new concepts and to broaden their understanding of older ideas.

Question → Explore and Conjecture → Prove

In general, this process begins with a **question** that helps you frame what you are looking for. For example, a question such as, "*What if the Möbius strip has 2 half-twists? What will happen when that strip is cut in half down the middle?*" can help start an investigation to find out what happens when the Möbius strip is slightly altered.

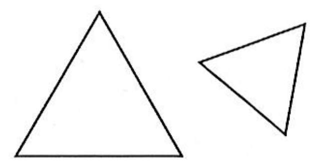
Once a question is asked, you can make an educated guess, called a **conjecture**. This is a mathematical statement that has not yet been proven.

Next, **exploration** begins. This part of the process may last awhile as you gather more information about the mathematical concept. For example, you may first have an idea about the diagonals of a rectangle, but as you draw and measure a rectangle on graph paper, you find out that your conjecture was incorrect. When this happens, you just experiment some more until you have a new conjecture to test.

When a conjecture seems to be true, the final step is to **prove** that the conjecture is always true. A proof is a convincing logical argument that uses definitions and previously proven conjectures in an organized sequence.

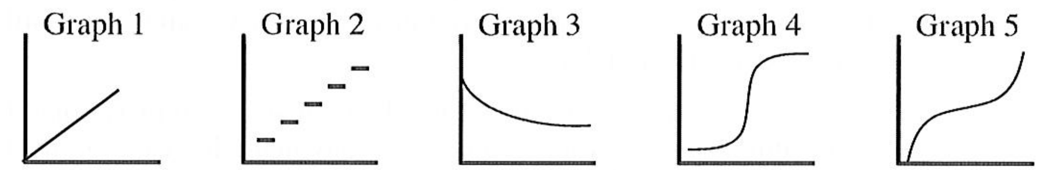
13. One part of inductive reasoning is asking mathematical questions. Assume your teacher is thinking of a shape and wants you to figure out what shape it is. Write down three questions you could ask your teacher to determine more about the shape.

14. The shapes are examples of **equilateral triangles.** How can you describe an equilateral triangle. Examine them and make at least two statements that seem to be true for all equilateral triangles. Draw another equilateral triangle in a different orientation.



Examples of Equilateral Triangles

15. Match each table of data with the most appropriate graph and briefly explain why it matches the data.



A. ______ Boiling water cooling down.

Time (min)	0	5	10	15	20	25
Temp (C)	100	89	80	72	65	59

B. ______ Cost of a phone call.

Time (min)	1	2	2.5	3	4	5	5.3	6
Cost (cents)	55	75	75	95	115	135	135	155

C. Growth of a baby in the womb.

Age (months)	1	2	3	4	5	6	7	8	9
Length (inches)	0.75	1.5	3	6.4	9.6	12	13.6	15.2	16.8

<u>**Video Example 1**</u>**. Find the next item in the pattern.**

A. A, E, I, . . .

B. 7, 14, 21, 28, . . .

C. 

1 Identifying a Pattern

Find the next item in each pattern.

A **Monday, Wednesday, Friday, …**
Alternating days of the week make up the pattern.
The next day is Sunday.

B **3, 6, 9, 12, 15, …**
Multiples of 3 make up the pattern. The next multiple is 18.

C $\leftarrow, \nwarrow, \uparrow, \ldots$
In this pattern, the figure rotates 45° clockwise each time.
The next figure is $\nearrow$.

Video Example 2. Complete the conjecture.

A. The product of two odd number is _____.

B. The number of circle parts formed by n diameters is _____.

2 Making a Conjecture

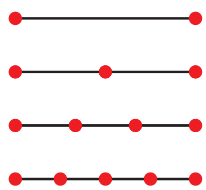
Complete each conjecture.

A **The product of an even number and an odd number is __?__.**
List some examples and look for a pattern.
$(2)(3) = 6$ $(2)(5) = 10$ $(4)(3) = 12$ $(4)(5) = 20$
The product of an even number and an odd number is even.

Complete each conjecture.

B **The number of segments formed by *n* collinear points is __?__.**

Draw a segment. Mark points on the segment, and count the number of individual segments formed. Be sure to include overlapping segments.



Points	Segments
2	1
3	$2 + 1 = 3$
4	$3 + 2 + 1 = 6$
5	$4 + 3 + 2 + 1 = 10$

The number of segments formed by *n* collinear points is the sum of the whole numbers less than *n*.

Video Example 3. In Beth's neighborhood, there are equal numbers of Mockingbirds, Cardinals, and Blue Jays. Each morning, for five days, Beth counted the number of birds at the feed outside her kitchen window. Make a conjecture based on the data.

	Day 1	Day 2	Day 3	Day 4	Day 5
Mockingbirds	5	4	5	3	3
Cardinals	7	10	12	8	7
Blue Jays	1	2	0	1	0

3 *Biology Application*

To learn about the migration behavior of California gray whales, biologists observed whales along two routes. For seven days they counted the numbers of whales seen along each route. Make a conjecture based on the data.

Numbers of Whales Each Day							
Direct Route	1	3	0	2	1	1	0
Shore Route	7	9	5	8	8	6	7

More whales were seen along the shore route each day. The data supports the conjecture that most California gray whales migrate along the shoreline.

Inductive Reasoning
1. Look for a pattern.
2. Make a conjecture.
3. Prove the conjecture or find a counterexample.

<u>Video Example 4</u>. Show that the conjecture is false by finding a counterexample.

A. For all positive numbers x, $x^2 \geq x$.

B. Four points on a plane always form a quadrilateral.

C. The temperature in Lubbock, Texas never exceeds 107°F during the spring months (March, April, & May).

Monthly High Temperatures (°F) in Lubbock, Texas											
Jan	Feb	Mar	Apr	May	Jun	Jul	Aug	Sep	Oct	Nov	Dec
83	84	91	97	109	111	114	103	103	102	91	82

4 Finding a Counterexample

Show that each conjecture is false by finding a counterexample.

A For all positive numbers n, $\frac{1}{n} \le n$.

Pick positive values for n and substitute them into the equation to see if the conjecture holds.

Let $n = 1$. Since $\frac{1}{n} = 1$ and $1 \le 1$, the conjecture holds.

Let $n = 2$. Since $\frac{1}{n} = \frac{1}{2}$ and $\frac{1}{2} \le 2$, the conjecture holds.

Let $n = \frac{1}{2}$. Since $\frac{1}{n} = \frac{1}{\frac{1}{2}} = 2$ and $2 \not\le \frac{1}{2}$, the conjecture is false.

$n = \frac{1}{2}$ is a counterexample.

B For any three points in a plane, there are three different lines that contain two of the points.

Draw three collinear points.

If the three points are collinear, the conjecture is false.

C The temperature in Abilene, Texas, never exceeds 100°F during the spring months (March, April, and May).

Monthly High Temperatures (°F) in Abilene, Texas

Jan	Feb	Mar	Apr	May	Jun	Jul	Aug	Sep	Oct	Nov	Dec
88	89	97	99	107	109	110	107	106	103	92	89

The temperature in May was 107°F, so the conjecture is false.

<u>Example 4</u>. Show that the conjecture is false by finding a counterexample.

A. For every integer n, n^3 is positive.

B. Two complementary angles are not congruent.

C. The monthly high temperature in Abilene is never below 90°F for two months in a row.

Monthly High Temperatures (°F) in Abilene, Texas											
Jan	Feb	Mar	Apr	May	Jun	Jul	Aug	Sep	Oct	Nov	Dec
88	89	97	99	107	109	110	107	106	103	92	89

<u>Guided Practice</u>: Show that the conjecture is false by finding a counterexample.

16. For any real number x, $x^2 \geq x$.

17. Supplementary angles are adjacent.

18. The radius of every planet in the solar system is less than 50,000 km.

Planets' Diameters (km)							
Mercury	Venus	Earth	Mars	Jupiter	Saturn	Uranus	Neptune
4880	12,100	12,800	6790	143,000	121,000	51,100	49,500

2-1 Using Inductive Reasoning to Make Conjectures *(p 77)* 11, 13-17, 19.